Two-nucleon transfer reactions uphold supersymmetry in atomic nuclei

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The spectroscopic strengths of two-nucleon transfer reactions constitute a stringent test for two-nucleon correlations in the nuclear wave functions. A set of closed analytic expressions for ratios of spectroscopic factors is derived in the framework of nuclear supersymmetry. These ratios are parameter independent and provide a direct test of the wave functions. A comparison between the recently measured ^{198}Hg$(\vec{d},\alpha)^{196}$Au reaction and the predictions from the nuclear quartet supersymmetry lends further support to the validity of supersymmetry in nuclear physics.

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The Interacting Boson Model (IBM) and its extensions have provided a bridge between single-particle and collective behavior in nuclei, based on the approximate bosonic nature of pairs of identical nucleons that dominate the dynamics of valence nucleons and that arise from the underlying nuclear forces. This is similar to the BCS theory of semi-conductors with its coupling of electrons to spin-zero Cooper pairs which leads to collective behavior and to superconductivity. The conceptual basis of the IBM has led to a unified description of the collective properties of medium and heavy mass even-even nuclei, pictured in this framework as belonging to transitional regions between various dynamical symmetries [1].

Odd-mass nuclei were also analyzed from this point of view, by incorporating the degrees of freedom of a single fermion [2]. In 1980, Iachello suggested a simultaneous description of even-even and odd-mass nuclei through the introduction of a superalgebra, with energy levels in both nuclei belonging to the same (super)multiplet [3]. In essence, this proposal is based on the fact that even-even nuclei behave as (composite) bosons while odd-mass ones behave as approximate fermions. At the appropriate length scales their states can be viewed as elementary. The bold and far-reaching idea was then put forward that both these nuclei can be embedded into a single conceptual framework, relating boson-boson and boson-fermion interactions in a precise way.

The concept of nuclear supersymmetry was extended in 1985 to include the neutron-proton degree of freedom [4]. In the new framework, a supermultiplet consists of an even-even, an odd-proton, an odd-neutron and an odd-odd nucleus. Spectroscopic studies of heavy odd-odd nuclei are very difficult due the high density of states. Almost 15 years after the prediction of the spectrum of the odd-odd nucleus by nuclear supersymmetry, it was shown experimentally that the observed spectrum of the nucleus ^{196}Au is amazingly close to the theoretical one [5].

However, transfer reactions provide a far more sensitive test of the wave functions than do energies. In particular, two-nucleon transfer reactions constitute a powerful tool in nuclear structure research [6]. In contrast to the better studied one-nucleon transfer reactions where the single-particle content of the states of the final nucleus is scrutinized, two-nucleon transfer reactions probe the structure of these states in a more subtle way through the exploration of two-nucleon correlations that may be present. The spectroscopic strengths of the two-nucleon transfer reaction depend on two factors: the similarity between the states in the initial and final nucleus which differ by two nucleons, and the correlation of the tranferred pair of nucleons. The information extracted through these reactions supply a challenging test of the calculated wave functions for any nuclear structure model.

The purpose of this Letter is to study two-nucleon transfer reactions in the $U_\nu(6/12) \otimes U_\pi(6/4)$ supersymmetry via selection rules and spectroscopic strengths and to test the predictions against the recent data on the ^{198}Hg$(\vec{d},\alpha)^{196}$Au reaction obtained in 2004 [7]. This reaction involves the transfer of a proton-neutron pair, and hence measures the neutron-proton correlation in the odd-odd nucleus.

$$\begin{array}{ccccc} & & & & {}^{198}_{80}\mathrm{Hg}_{118} \\ & & & \swarrow & \\ {}^{195}_{79}\mathrm{Au}_{116} & \leftrightarrow & {}^{196}_{79}\mathrm{Au}_{117} & & \\ \updownarrow & & \updownarrow & & \\ {}^{194}_{78}\mathrm{Pt}_{116} & \leftrightarrow & {}^{195}_{78}\mathrm{Pt}_{117} & & \end{array}$$

The final odd-odd nucleus ^{196}Au has been suggested as a member of a supersymmetric quartet of nuclei [4] of the $U_\nu(6/12) \otimes U_\pi(6/4)$ dynamical supersymmetry in which the odd neutron is allowed to occupy the $2\nu f_{5/2}$, $3\nu p_{3/2}$ and $3\nu p_{1/2}$ orbits of the 82-126 shell and the odd proton the $2\pi d_{3/2}$ level of the 50-82 shell. It incorporates both the $U(6/4)$ scheme [3] for the even-even and odd-proton nuclei and the $U(6/12)$ scheme [8] for the even-even and odd-neutron nuclei in the Pt-Au mass region. In this extended scheme which includes the neutron-proton degree of freedom, the four nuclei ^{194,195}Pt and ^{195,196}Au form a supersymmetric quartet.

The relevant subgroup chain of $U(6/12)_\nu \otimes U(6/4)_\pi$ for the Pt and Au nuclei is given by [4]

$$\begin{aligned}
U(6/12)_\nu \otimes & \; U(6/4)_\pi \\
\supset & \; U^{B_\nu}(6) \otimes U^{F_\nu}(12) \otimes U^{B_\pi}(6) \otimes U^{F_\pi}(4) \\
\supset & \; U^{B}(6) \otimes U^{F_\nu}(6) \otimes U^{F_\nu}(2) \otimes U^{F_\pi}(4) \\
\supset & \; U^{BF_\nu}(6) \otimes U^{F_\nu}(2) \otimes U^{F_\pi}(4) \\
\supset & \; SO^{BF_\nu}(6) \otimes U^{F_\nu}(2) \otimes SU^{F_\pi}(4) \\
\supset & \; Spin(6) \otimes U^{F_\nu}(2) \\
\supset & \; Spin(5) \otimes U^{F_\nu}(2) \\
\supset & \; Spin(3) \otimes SU^{F_\nu}(2) \\
\supset & \; SU(2) \; . \qquad (1)
\end{aligned}$$

In a dynamical supersymmetry the Hamiltonian is expressed in terms of Casimir invariants of the groups appearing in the chain of Eq. (1) leading to a closed form for the energy spectrum and a direct correlation between the wave functions of the four nuclei that make up the quartet.

For simplicity, the ground state wave function of the initial nucleus ^{198}Hg is taken to be that of the $SO(6)$ limit of the IBM [1]

$$\left|^{198}\mathrm{Hg}\right\rangle \;=\; |[N_\nu],[N_\pi];[N],(N,0,0),(0,0),0\rangle \;, \qquad (2)$$

where $N = N_\nu + N_\pi$ is the total number of bosons. Its parity is positive. In the nuclear supersymmetry classification scheme, the wave functions of the final nucleus ^{196}Au have a more complicated structure since they consist of a bosonic part characterized by the same number of proton and neutron bosons as the initial nucleus ^{198}Hg, and a fermionic part for the proton orbit $j_\pi = 3/2$ and the neutron orbits $j_\nu = 1/2,\ 3/2,\ 5/2$ characterized by the labels

$$\begin{aligned}
\pi \;&:\; \left|(\frac{1}{2},\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),j_\pi=\frac{3}{2}\right\rangle \;, \\
\nu \;&:\; \left|[1],(1,0,0),(\tau,0),2\tau,\frac{1}{2};j_\nu\right\rangle \;, \qquad (3)
\end{aligned}$$

with $\tau = 0,1$. The labels of the proton orbital are those of the spinor representations of $SO(6)$ and $SO(5)$ [3]. The neutron orbitals are decomposed into a pseudo-orbital part $k = 2\tau$ (with $\tau = 0,1$) and a spin part $s = 1/2$. The pseudo-orbital angular momenta span the six-dimensional representations [1] and $(1,0,0)$ of $U(6)$ and $SO(6)$, respectively, which contain $(\tau,0) = (0,0)$ and $(1,0)$ of $SO(5)$ [8]. The wave functions of ^{196}Au are obtained by combining those of the even-even nucleus of Eq. (2) with the single-particle wave functions of Eq. (3) into

$$\begin{aligned}
\left|^{196}\mathrm{Au}\right\rangle \;=\; & |[N_\nu],[N_\pi];[N],[1]_\nu;[N_1,N_2],(\Sigma_1,\Sigma_2,0), \\
& \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)_\pi;(\sigma_1,\sigma_2,\sigma_3),(\tau_1,\tau_2),J',\frac{1}{2};J\right\rangle \;, \qquad (4)
\end{aligned}$$

Table I: States in ^{196}Au that can be excited from the ground state in ^{198}Hg by the tensor operators T_1, T_2 and T_3 of Eq. (6) for two-nucleon transfer reactions.

$(\sigma_1,\sigma_2,\sigma_3)$	(τ_1,τ_2)	T_1	T_2	T_3
$(N\pm\frac{3}{2},\frac{1}{2},\pm\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$		$\surd$	
	$(\frac{3}{2},\frac{1}{2})$			$\surd$
$(N\pm\frac{1}{2},\frac{1}{2},\mp\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$\surd$	$\surd$	
	$(\frac{3}{2},\frac{1}{2})$			$\surd$
$(N\pm\frac{1}{2},\frac{3}{2},\pm\frac{1}{2})$	$(\frac{3}{2},\frac{1}{2})$			$\surd$

where the labels denote the irreducible representations of the groups appearing in Eq. (1) [4]. Due to the choice of the single-particle orbits, the parity of the states in Eq. (4) is odd.

In first order, the form of the two-nucleon transfer operator for the $(\vec{d},\alpha)$ reaction is simply given by

$$(a^\dagger_{j_\nu} a^\dagger_{j_\pi})^{(\lambda)} \;. \qquad (5)$$

In the IBM and its extensions the number of particles corresponds to the number of valence nucleons in the first half of the major shell and to holes in the second half. Therefore, in the present appliciation the creation operators in Eq. (5) correspond to holes. The selection rules can be determined from the tensorial character of the proton and neutron orbits of Eq. (3). As a result, the transfer operator of Eq. (5) can be expanded in terms of three tensor operators $T^{(s_1,s_2,s_3)}_{(t_1,t_2),J',1/2;J}$ where (s_1,s_2,s_3) denotes the tensorial character under $Spin(6)$, (t_1,t_2) under $Spin(5)$, J' under $Spin(2)$ and J under $SU(2)$

$$\begin{aligned}
T_1 \;&=\; T^{(\frac{1}{2},\frac{1}{2},-\frac{1}{2})}_{(\frac{1}{2},\frac{1}{2}),J',\frac{1}{2};J} \;, \\
T_2 \;&=\; T^{(\frac{3}{2},\frac{1}{2},\frac{1}{2})}_{(\frac{1}{2},\frac{1}{2}),J',\frac{1}{2};J} \;, \\
T_3 \;&=\; T^{(\frac{3}{2},\frac{1}{2},\frac{1}{2})}_{(\frac{3}{2},\frac{1}{2}),J',\frac{1}{2};J} \;. \qquad (6)
\end{aligned}$$

The allowed values of $(\sigma_1,\sigma_2,\sigma_3)$ and (τ_1,τ_2) of the wave functions of ^{196}Au are presented in Table I.

The matrix elements of the transfer operator of Eq. (5) can be derived in closed analytic form by using standard tensor algebra

$$\begin{aligned}
&\left\langle^{196}\mathrm{Au}\right\| (a^\dagger_{j_\nu} a^\dagger_{j_\pi})^{(\lambda)} \left\|^{198}\mathrm{Hg}\right\rangle \\
&\quad=\; \delta_{\lambda,J}(-)^{j_\nu+J'}\,\hat{J}\hat{j}_\nu\hat{J}' \begin{Bmatrix} 2\tau & \frac{1}{2} & j_\nu \\ J & \frac{3}{2} & J' \end{Bmatrix}
\end{aligned}$$

$$\left\langle \begin{array}{cc|c} [N] & [1] & [N_1,N_2] \\ & & \\ (N,0,0) & (1,0,0) & (\Sigma_1,\Sigma_2,0) \end{array} \right\rangle$$
$$\left\langle \begin{array}{cc|c} (\Sigma_1,\Sigma_2,0) & (\frac{1}{2},\frac{1}{2},\frac{1}{2}) & (\sigma_1,\sigma_2,\sigma_3) \\ & & \\ (\tau,0),2\tau & (\frac{1}{2},\frac{1}{2}),\frac{3}{2} & (\tau_1,\tau_2),J' \end{array} \right\rangle$$
$$\left\langle \begin{array}{cc|c} (N,0,0) & (1,0,0) & (\Sigma_1,\Sigma_2,0) \\ & & \\ (0,0),0 & (\tau,0),2\tau & (\tau,0),2\tau \end{array} \right\rangle , \qquad (7)$$

with $\hat{L} = \sqrt{2L+1}$. The coefficients in brackets $< \mid >$ denote isoscalar factors [9, 10, 11, 12, 13]. The explicit results will be published in a separate article [13].

The ^{198}Hg$(\vec{d},\alpha)^{196}$Au reaction is characterized by the transfer of a correlated neutron-proton pair with spin $S = 1$. Since the angular momentum of the ground state of ^{198}Hg is zero, the transfered total angular momentum λ is equal to the angular momentum J of the final state of ^{196}Au. Thus for each value of $\lambda = J$ there are three different transfers corresponding to $L = J - 1$, J and $J + 1$. Since the initial and final states have opposite parity, parity conservation limits the allowed values of L to be odd. The transferred angular momentum and parity of the two-nucleon transfer operator in Eq. (5) is $J^\pi = 0^-, 1^-, 2^-, 3^-, 4^-$. The L transfer with total angular momentum J is denoted as L_J. The seven possibilities are P_0, P_1, P_2, F_2, F_3, F_4 and H_4.

The experimental values of the spectroscopic strengths G_{LJ} for the transfer of a neutron-proton pair were determined from the measurement of the angular distributions of the differential cross section and the analyzing power of the ^{198}Hg$(\vec{d},\alpha)^{196}$Au reaction [7]. Theoretically, the spectroscopic strengths can be written as

$$G_{LJ} = \left| \sum_{j_\nu j_\pi} g^{LJ}_{j_\nu j_\pi} \langle^{196}\mathrm{Au} \| \, (a^\dagger_{j_\nu} a^\dagger_{j_\pi})^{(\lambda)} \, \|^{198}\mathrm{Hg} \rangle \right|^2 , \qquad (8)$$

where the coefficients $g^{LJ}_{j_\nu j_\pi}$ contain factors that arise from the reaction mechanism for two-nucleon transfer reactions, such as a $9-j$ symbol for a change of angular momentum coupling from jj to LS coupling and a Talmi-Moshinksy bracket for the transformation to relative and center-of-mass coordinates of the transferred nucleons [6]. The nuclear structure part is contained in the reduced matrix elements of Eq. (7).

In order to compare with experimental data we calculate for each combination of L_J the relative strengths from the ratio

$$R_{LJ} = G_{LJ}/G^{\mathrm{ref}}_{LJ} , \qquad (9)$$

where G^{ref}_{LJ} is the spectroscopic strength of the reference state for a particular L_J transfer.

The tensorial character of the transfer operator of Eq. (5) shows that in the supersymmetry scheme only states in ^{196}Au with $(\tau_1,\tau_2) = (\frac{3}{2},\frac{1}{2})$ and $(\frac{1}{2},\frac{1}{2})$ can be excited (see Table I). The angular momentum states belonging to $(\frac{3}{2},\frac{1}{2})$ have $J' = \frac{1}{2}, \frac{5}{2}, \frac{7}{2}$ and $J = J' \pm \frac{1}{2}$. Table I shows that they can only be excited by the tensor operator T_3. Therefore, the ratios of spectroscopic strengths to these states provide a direct test of the nuclear wave functions, since they do not depend on the coefficients $g_{j_\nu j_\pi}$, but only on the nuclear structure part, *i.e.* the reduced matrix elements of T_3. If we take the states with $[N_1,N_2] = [5,1]$, $(\Sigma_1,\Sigma_2,0) = (5,1,0)$ and $(\sigma_1,\sigma_2,\sigma_3) = (\frac{11}{2},\frac{3}{2},\frac{1}{2})$ as reference states, we find

$$R_{LJ} = \frac{N+4}{15N} , \qquad (10)$$

for $[5,1]$, $(5,1,0)$, $(\frac{11}{2},\frac{1}{2},-\frac{1}{2})$ and

$$R_{LJ} = \frac{2(N+4)(N+6)}{15N(N+3)} , \qquad (11)$$

for $[6,0]$, $(6,0,0)$, $(\frac{13}{2},\frac{1}{2},\frac{1}{2})$. The numerical values are 0.12 and 0.33, respectively (for $N = 5$). The angular momentum states belonging to $(\frac{1}{2},\frac{1}{2})$ have $J' = \frac{3}{2}$ and $J = J' \pm \frac{1}{2}$. Table I shows that they can be excited by the tensor operators T_1 and T_2. For these states the ratios R_{LJ} depend both on the reaction and structure part.

In Fig. 1 we show the experimental and calculated ratios R_{LJ}. The reference states can easily be identified since they are normalized to one. The P_0, P_1, F_3, F_4 and H_4 transfers are normalized to the states assigned as $[5,1]$, $(5,1,0)$, $(\frac{11}{2},\frac{3}{2},\frac{1}{2})$, $(\frac{3}{2},\frac{1}{2})$, whereas the P_2 and F_2 transfers to the $[6,0]$, $(6,0,0)$, $(\frac{13}{2},\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2})$ states.

We observe in general that there is good overall agreement between the experimental and theoretical values, especially if we take into account the simple form of the operator in the calculation of the two-nucleon transfer reaction intensities. We can see that large ratios are well reproduced except for one related to a 4^- state and that all small ratios are consistent with the experimental data.

These results have led us to a change in the assignment used previously for the 2^- state at 166.6(5) keV [14]. It is now associated to the theoretical state with labels $[5,1]$, $(5,1,0)$, $(\frac{11}{2},\frac{1}{2},-\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2})$.

In conclusion, we have studied the two-nucleon pickup reaction ^{198}Hg$(\vec{d},\alpha)^{196}$Au as a test of the nuclear supersymmetry scheme proposed for the Pt-Au region [4, 5]. Two-nucleon transfer reactions $(\vec{d},\alpha)$ not only offer a powerful tool to help establish the spin and parity assignments of the energy levels in the odd-odd nucleus ^{196}Au [7], but also provide a sensitive test of neutron-proton correlations in the wave functions. The symmetry structure of the model gives rise to selection rules and parameter independent predictions of ratios of spectroscopic strenghts which only depend on the nuclear structure part, and not on factors that arise from the kinematical

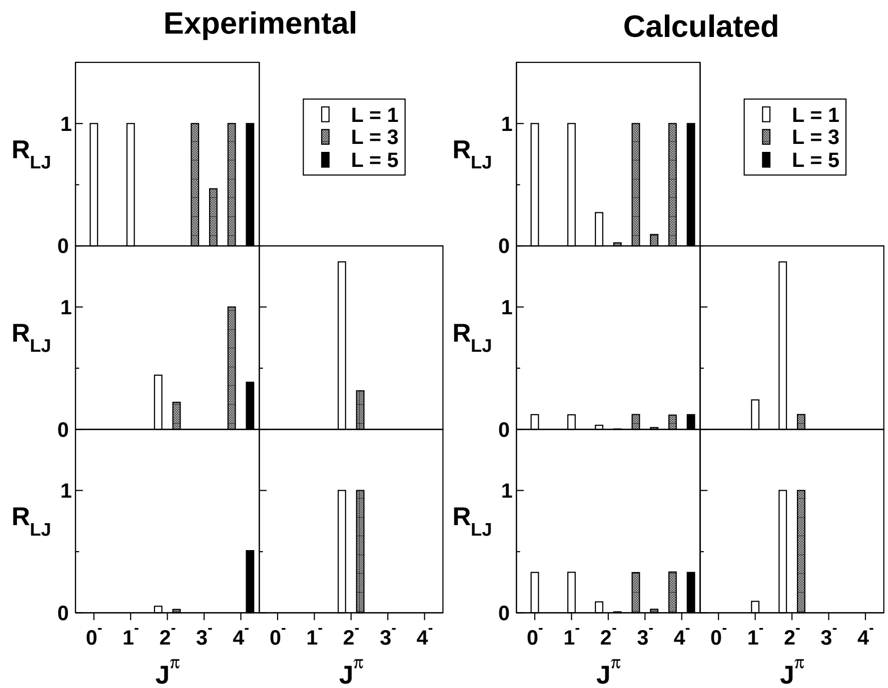

Figure 1: Ratios of spectroscopic strengths. The wave functions of the final nucleus ^{196}Au are given by Eq. (4). The two columns in each frame correspond to states with $Spin(5)$ labels $(\tau_1,\tau_2)=(\frac{3}{2},\frac{1}{2})$ and $(\frac{1}{2},\frac{1}{2})$, respectively. The rows are characterized by the labels $[N_1,N_2]$, $(\Sigma_1,\Sigma_2,0)$, $(\sigma_1,\sigma_2,\sigma_3)$. From bottom to top we have (i) $[6,0]$, $(6,0,0)$, $(\frac{13}{2},\frac{1}{2},\frac{1}{2})$, (ii) $[5,1]$, $(5,1,0)$, $(\frac{11}{2},\frac{1}{2},-\frac{1}{2})$ and (iii) $[5,1]$, $(5,1,0)$, $(\frac{11}{2},\frac{3}{2},\frac{1}{2})$.

part. A comparison with experimental data shows a surprisingly good overall agreement with the predictions of the supersymmetry scheme and, in this way, lends further support to the validity of the supersymmetry scheme in atomic nuclei, especially in the Pt-Au mass region. More tests involving the ^{194}Pt and ^{196}Au nuclei [13, 15] as well as nuclei in other parts of the nuclear mass table [16] are currently underway.

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